

# Technologies for Non-Circular Gear Generation and Manufacture

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## ABSTRACT

*Non-circular gears, not very popular in old gear industry, have increasingly become nowadays a challenge for both mathematicians and engineers. The development of softwares and modern computer-controlled machine tools enables the complex non-circular gear design and manufacture to be feasible tasks.*

*The paper presents a short overview on non-circular gear generation methods, based on the enveloping theory, analytical approximations of tooth profile and simulations of gear generation. The various shapes of the gear centrodes lead to a careful selection of tools and cutting kinematics; a brief review of the topic is also presented, highlighting the latest approaches in non-circular gear manufacture.*

**KEYWORDS:** non-circular gears, non-circular gear centrode, non-circular gear generation, non-circular gear manufacture.

## 1. Introduction

Many mechanical applications require mechanisms able to produce variable motions, according to specific laws, i.e., cams, linkages or non-circular gears. Cams and linkages were more popular in the past, mainly because of the design complexity and high production prices of non-circular gears. This situation has been changed over time due to the development of modern production methods that use computer-controlled machine tools, as well as to the development of new mathematical models.

Nowaday, the non-circular gears have started to replace cams and linkage in those mechanisms that perform a purely mechanical control on the input/output relationship [1]. They are used for the generation of a nonlinear relation between the pinion and the driven gear, through the shape of their closed or open centrodes, with elliptical, oval, squared or other shapes, consisting of both convex and concave segments [2].

While Freudenstein and Chen proposed the use of non-circular sprockets for bicycles [3], Emura and Arakawa used non-circular gears to make a new steering mechanism, capable of turning a carrier with a small radius [4]. For the same purpose, i.e. to design a steering mechanism, but this time for automobiles, Dooner used noncircular gears along with an eight-link mechanism [5]. The same author used these gears to reduce unwanted torque and speed fluctuations that may exist in rotating shafts [6] and together with Barkah and Shafiq generated a 3D mesh to determine

their static stress [7]. A new press concept, that used noncircular gears in the drive mechanism, was proposed by Doege [8]. Librovich designed a multistage rotary vane engine incorporating the same category of gears [9], while Kowalczyk and Urbanek used them in textile industry [10]. Noncircular gears were also used for polishing machines, by Liu et al. [11], for the generation of a motion law imitating the blood pumping of the heart, by Ottaviano et al. [12], and for generating an optimal trajectory in a rice transplanter, by Bae and Yang [13]. A coupler point can be guided along a prescribed planar trajectory with the help of a five-bar linkage, integrated by a pair of non-circular gears, as shown by D. Mundo and G. Gatti [14]. The same subject is studied by K.-H. Modler et al., but they present a general synthesis method to design a transmission function, using five-links geared linkages [15].

Most of the scientists have focused their attention on computer-aided design and on analyzing the kinematic relations between the pitch curves of non-circular gears. The first step in non-circular gear modeling is the pitch curves design, leading to various methods of designing the teeth along. Tong and Yang designed identical noncircular pitch curves of an unlimited profile varieties and any number of lobes, by using an algorithm based on the criteria for two identical rigid bodies to engage in conjugate rolling [16]. Yang also used deviation functions to reshape the original pitch pairs, obtaining in this way the desired profiles [17]. Lozzi generated non-circular gear base curves by graphic construction and

implementation by computer algorithms [18], while Riaza et al. obtained the base curves as the geometrical locus of the singular points in the involute tooth profile [19]. Another method of modeling noncircular pitch curves was used by Tsay et al, Liu et. al [20,21] and it involved the Fourier series for the approximation of the desired pitch outlines. The mathematical model of the teeth was based on this approximation. Using a function for the pressure angle, Jing described a general mathematical model for the tooth profiles of planar gears [22]. This method is applicable to any type of gear geometry, making it easy to obtain some desired geometrical and mechanical properties of the gears. Jia Yan et al. proposed a method for the design of noncircular multilobe internal pitch curves [23], while variable transmissions with intersecting axes was the subject of a study made by Jiqiang Xia et al. [24]. Bair proposed mathematical models for the generation of circular-arc elliptical gears with rack cutters and shaper cutters taking into consideration the undercutting phenomenon that may appear [25-29], while Mundo and Danieli [30,31] used a constant pressure angle to obtain noncircular gears with a greater contact ratio between the teeth.

## 2. Methods of Generating Non-Circular Gear

Currently, the existing methods for non-circular gear generation derive from the main algorithm presented in figure 1.

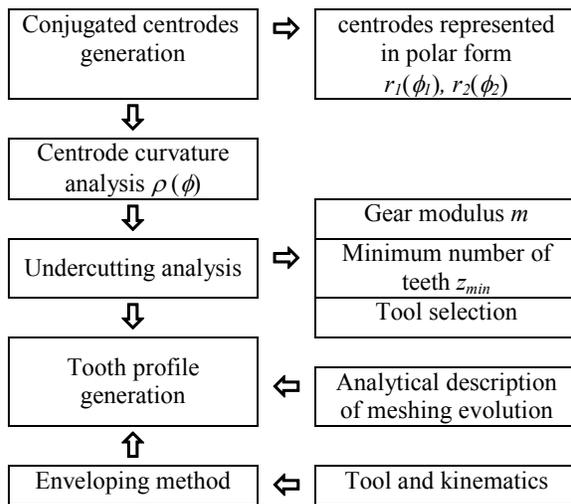


Fig. 1. General procedure for non-circular gear generation

### 2.1. Non-circular gear generation using the enveloping theory

The first who came up with the brilliant idea of using the same tools for the manufacturing of non-circular gears, as for the production of circular ones, was F. Litvin who developed the enveloping theory [32-35]. The enveloping theory was based on obtaining tooth surface as the envelope of the family of tool surfaces and the main features were the following:

- the noncircular gear could be manufactured with the same tools as the circular ones;
- the conjugated tooth shape could be obtained by the imaginary rolling of the tool centrode over the given gear, considering a proper kinematics.

The generation of noncircular gears is presented in fig. 2, where centrodes 1 and 2 of the noncircular gears are meshing with the rack cutter 3. The centrode of the rack cutter is represented by a straight line  $t-t$  that is a common tangent to centrode 1 and 2 and rolls over the two centrodes. The movement of the rack cutter was obtained by its translation along tangent  $t-t$  and its rotation about the instantaneous center of rotation  $I$ . Tooth surfaces of the two gears and of the rack cutter are in mesh at the same time, and the two gears have conjugated shapes.

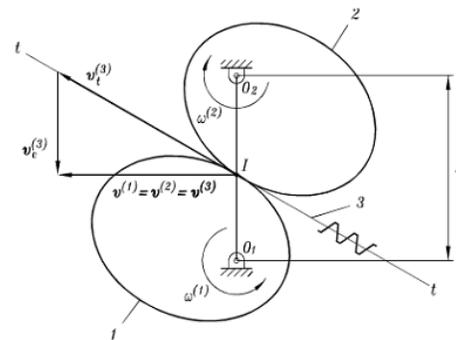


Fig.2. Generation of noncircular gears 1 and 2 by rack cutter 3 [33]

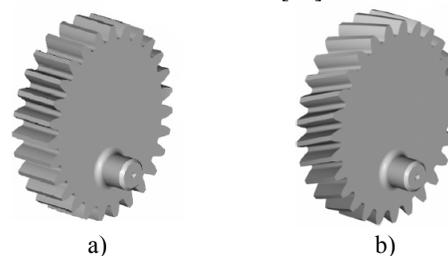


Fig. 3. Elliptical gears with straight (a) and helical teeth (b) [33]

Figure 3 illustrates two examples of non-circular gears generated by Litvin using his theory.

Chang and Tsay proposed an alternative method for the generation of noncircular gears, that considers the inverse mechanism relation and the

equation of meshing to determine the mathematical model of a complete noncircular gear profile generated by a shaper cutter [37]. The noncircular gears generated by this method, include working region, fillet, top land surfaces and backlash. An example of noncircular gear generated using the inverse mechanism relation and the equation of meshing is illustrated in fig. 4.

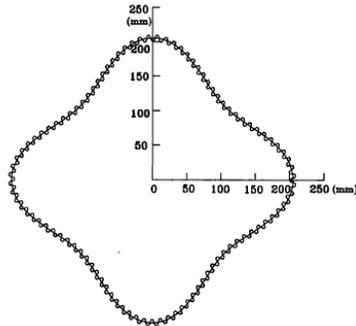


Fig. 4. Noncircular gear generated using the inverse mechanism relation and the equation of meshing [37]

2.2. Noncircular gear generation using Bezier curves

Riaza et al. developed a method for defining the relationship between the angular coordinates of N-lobe noncircular gears using Bezier nonparametric curves [38]. The conjugate pitch curves were obtained by defining the displacement law and using an analytical method that transformed the pinion angular coordinates in input variables (fig. 5).

The advantage of Bezier curves was that they could produce the displacement laws of noncircular gears. Thus, a non-circular gear train that satisfied the given relationship between the pinion and driven wheel rotation angles, respectively, was generated. The flexibility of Bezier curves was limited by two aspects: the resulting polynomial degree was fixed by the number of polygon vertices and the only way to reduce it was to reduce the number of vertices; the second aspect was that changing one vertex of the curve, the entire curve was changed and so local changes could not be further made.

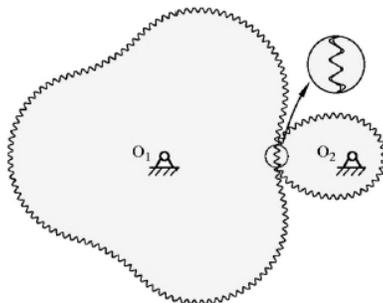


Fig. 5. Noncircular gears with a displacement law given by a seven-degree Bezier curve with  $C^3$  continuity

2.3. Noncircular gear generation by simulating the shaping process

JianGang Li et. al. proposed a simple and accurate numerical method for generating the tooth profile of a non-circular gear [39]. Unlike other methods, this one was based directly on the gear shaping process and did not need solving complicated equations. The generation of the tooth profile was obtained from the boundary produced when the cutter profile was plotted on the gear transverse plane. The relative position of the cutter profile was given by the noncircular gear pitch line, by the shaper cutter parameters and by the data obtained in the shaping process. This method is more efficient and accurate, especially for the non-circular gears with complex geometry. The authors also considered pointed teeth, undercutting and fillet interference.

Figure 6. shows the kinematic relationship between the generated noncircular gear and the shaper cutter. The tool rotates about its center  $O_g$  and in the same time translates along a curve keeping the shaper centre and pitch line of the noncircular gear in tangency.

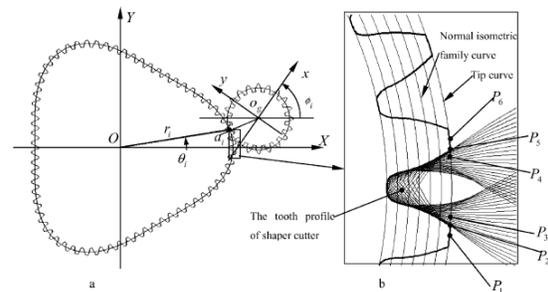


Fig. 6. Generation of a noncircular gear by a shaper cutter [39]  $\theta_i$  – polar angle of the pitch line at the point  $a_p$ ,  $\theta_i$  – shaper cutter’s rotation angle,  $P_1, P_2 \dots P_6$  – points situated on the tooth profile

3. Technologies for Non-Circular Gears Manufacture

Non-circular gear can be manufacture either by copying or rolling, considering the proper kinematics for tool and gear blank. The tooth profile can be generated by modular disk, rack and worm mills, disk cutters and racks.

During the manufacturing process, it is very important to select the proper geometrical parameters for the cutting tools; otherwise undercutting may occur and the tooth thickness near the fillet will be decreased, leading to a decreased load capacity.

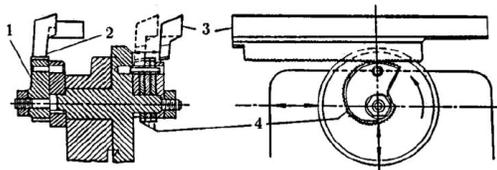
3.1. Generation by copying

Using copies is easy to apply and does not assume complex tool, nevertheless it presents several disadvantages:

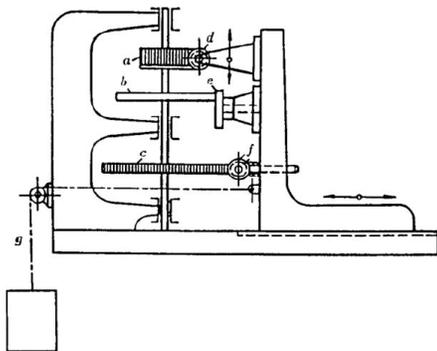
- a special copy is requested for every gear geometry, and the manufacturing and installation errors of the copy will be transmitted to the manufactured gears;

- gears with internal crowns and those with convex-concave segments cannot be cut;
- many passes are necessary.

Examples of non-circular gear manufacture, by copying, are presented in fig. 7-8. Figure 8 illustrates Fellow’s approach (1924), a master noncircular gear was meshed with a master rack for the generation of noncircular gears. Figure 8 illustrates Bopp and Reuther’s approach (1938), in which t, a noncircular master worm gear, was meshed with a worm identical to the hob.



**Fig. 7.** Noncircular gear generation by meshing a master noncircular gear with a master rack  
1-master noncircular gear; 2-master rack; 3-rack cutter; 4-noncircular gear being generated [33]



**Fig. 8.** Noncircular gear generation by meshing a noncircular master worm gear with a worm  
a-noncircular gear being generated; b-cam; c-master worm gear; d-hob; e-follower; f-worm [33]

**3.2. Generation by rolling**

Generation by rolling, using a milling tool, is more accurate and more universal. During the tooth cutting process there are three possible combinations of motion (fig. 9):

a) the mill is rotated at constant speed while the gear is rotated at variable speed and translated parallel to the common normal NN, in a nonuniform manner (fig 9.a);

b) the mill is rotated at constant speed and is translated along AA, at variable speed, while the gear is rotated with a constant speed and translated parallel to NN, in a nonuniform manner (fig 9.b);

c) the mill is rotated at a constant speed and translated along AA, at a variable speed, while the

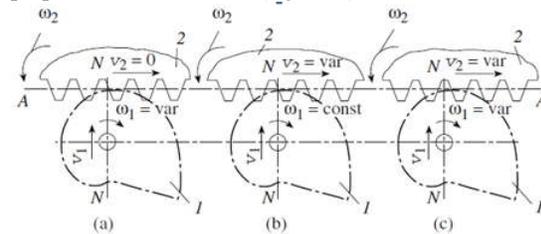
gear is rotated with a variable speed and translated parallel to NN, in a nonuniform manner (fig 9.c).

Gears with convex-concave centrodes or internal teeth cannot be machined by milling, but they can be manufactured with a slotting machine. The tooth profile is generated by rolling a slotting tool over their centrodes. Like in the milling process, here too there are three possible combinations of movements (fig. 10):

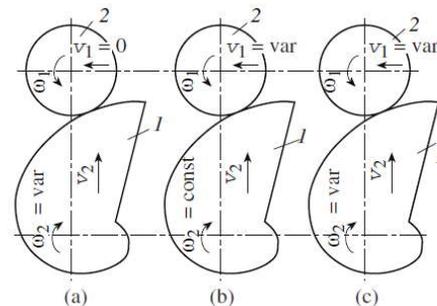
a) the tool is rotated at a constant speed and the gear is rotated at variable speed and translated, at a variable speed, along the line connecting the two centers (fig. 10.a);

b) the tool and the gear are rotated uniformly and translated at a variable speed into perpendicular directions (fig. 10.b);

c) the tool is rotated at a constant speed and the gear is rotated at a variable speed. The tool and the gear are also translated at variable speeds, in perpendicular directions (fig. 10.c).



**Fig. 9.** Noncircular gear cutting using a helical mill [2]



**Fig. 10.** Noncircular gear cutting using a slotting tool [2]

The non-circular gear can also be made by manufacture using rack-cutters, shaper cutters, hobs. Also, other specific technologies are mentioned in the references, i.e., electron discharge machining (fig. 11) or by rapid prototyping.



Fig. 11. Noncircular gear manufacturing using electron discharge machining [39]

#### 4. Conclusions

In the past, non-circular gears were not widely spread in gear industry due to their complex design and lack of understanding of their properties. Moreover, the manufacturing process of non-circular gears was not a simple task at all.

The situation changed over time, with the development of modern computer-controlled machine tools and new mathematical models for noncircular gear generation. Many scientists have focused their attention on finding new manufacturing methods, on improving the existing models and finding solutions for incorporating noncircular gears into various mechanisms.

The first step in non-circular gear generation is the design of conjugated centrodes; few approaches, generally based on rolling theory, are mentioned in the literature. The real challenge has been proved to be the further step, i.e. the generation of the tooth profile on various centrode shapes. Methods such as enveloping theory, analytical developments and virtual gear manufacture have been developed, bearing in mind the basic condition – avoidance of undercutting.

The non-circular gear manufacture was developed by copying or rolling, using special designed equipment and kinematics. Electron discharge machining and rapid prototyping are also technologies mentioned in the references.

The increased potential in non-circular gear design and manufacture explains the relatively new and continuous researches on these non-conventional gears performance.

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### Tehnologii de generare și prelucrare a roților dinate necirculare

#### Rezumat

Rotile dinate necirculare, mai puțin cunoscute în trecut, în industria angrenajelor, au devenit o adevărată provocare, în prezent, atât pentru matematicieni cât și pentru ingineri. Dezvoltarea aplicațiilor software și a mașinilor-unelte moderne, cu comanda numerică, a facilitat procesul complex de proiectare și prelucrare a roților dinate necirculare.

Lucrarea prezintă o scurtă trecere în revista a metodelor de generare a roților dinate necirculare, bazate pe teoria înfășurării suprafețelor, pe metode matematice de aproximare a profilului dintelui și pe simulări ale prelucrării danturii. Forma variată a curbilor de divizare ale roților implică o atenție deosebită privind alegerea sculei de prelucrare și a cinematicii procesului; un scurt rezumat al cercetărilor în domeniu este prezentat, evidențiindu-se metodele actuale de prelucrare a roților dinate necirculare.